W1472 – Grundzüge der Statistik II
Formelsammlung

Deskriptive Statistik

Mittelwerte:

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{k} x_i n_i \;=\; \frac{1}{n}\sum_{i=1}^{n} x_i \;= \sum_{i=1}^{k} x_i h_i$$

$$\overline{x+y} = \overline{x} + \overline{y} \qquad\qquad \overline{a+b\cdot x} = a + b\cdot\overline{x}$$

Streuungsmaße:

$$s_X^2 \;=\; \frac{1}{n}\sum_{i=1}^{k}(x_i-\overline{x})^2 n_i \;=\; \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \overline{x}^2 \;=\; \frac{1}{n}\sum_{i=1}^{k} x_i^2 n_i - \overline{x}^2$$

$$s_X^2 \;=\; \frac{1}{n}\sum_{i=1}^{k}\sum_{j=1}^{n_i}\left(x_{ij}-\overline{x}_i\right)^2 + \;\frac{1}{n}\sum_{i=1}^{k}(\overline{x}_i-\overline{x})^2 n_i \;=\; \frac{1}{n}\sum_{i=1}^{k} s_{Xi}^2 n_i + \;\frac{1}{n}\sum_{i=1}^{k}\left(\overline{x}_i-\overline{x}\right)^2 n_i \;=\; s_{Xint}^2 + s_{Xext}^2$$

$$s_{a+bx}^2 = b^2 s_x^2$$

$$s_{ax+by}^2 = a^2 s_x^2 + b^2 s_y^2 + 2ab\cdot c_{XY}$$

$$s = +\sqrt{s^2} \qquad\qquad VC = \frac{s}{\overline{x}}$$

Bedingte Häufigkeitsverteilung:

$$h(y_j \mid x_i) = \frac{n(x_i; y_j)}{n(x_i)} = \frac{n_{ij}}{n_i} \qquad\qquad h(x_i \mid y_j) = \frac{n(x_i; y_j)}{n(y_j)} = \frac{n_{ij}}{n_j}$$

Kovarianz:

$$c_{XY} = \operatorname{cov}(X,Y) = \frac{1}{n}\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y}) = \overline{xy} - \overline{x}\cdot\overline{y} = \frac{1}{n}\sum_{i=1}^{k}\sum_{j=1}^{l}\left(x_i-\overline{x}\right)\left(y_j-\overline{y}\right)n_{ij}$$

Regressionskoeffizienten nach KQ - Methode:

$$\sum_{i=1}^{n} y_i = na + b\sum_{i=1}^{n} x_i \qquad\qquad \sum_{i=1}^{n} x_i y_i = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2$$

$$b = \frac{\sum_{i=1}^{n}(x_i-\overline{x})\cdot(y_i-\overline{y})}{\sum_{i=1}^{n}(x_i-\overline{x})^2} = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} = \frac{c_{XY}}{s_X^2} \qquad\qquad a = \overline{y} - b\overline{x}$$

Zusammenhangsmaße:

$$QK = \sum_{i=1}^{k}\sum_{j=1}^{l}\frac{(n_{ij} - E_{ij})^2}{E_{ij}} \qquad KK = \sqrt{\frac{QK}{QK+n}} \qquad KK^* = \frac{KK}{KK_{\max}} = \sqrt{\frac{QK \cdot m}{(QK+n)\cdot(m-1)}}$$

$$KK_{\max} = \sqrt{\frac{m-1}{m}} \qquad m = \min\{k,l\}$$

$$r_{XY} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})\cdot(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}} \qquad B = R^2 = r_{XY}^2 = b \cdot b'$$

Indexrechnung:

$$P_{0,t}^{L} = \frac{\sum_{i=1}^{n} p_t(i)\cdot q_0(i)}{\sum_{i=1}^{n} p_0(i)\cdot q_0(i)} \qquad Q_{0,t}^{L} = \frac{\sum_{i=1}^{n} p_0(i)\cdot q_t(i)}{\sum_{i=1}^{n} p_0(i)\cdot q_0(i)}$$

$$P_{0,t}^{P} = \frac{\sum_{i=1}^{n} p_t(i)\cdot q_t(i)}{\sum_{i=1}^{n} p_0(i)\cdot q_t(i)} \qquad Q_{0,t}^{P} = \frac{\sum_{i=1}^{n} p_t(i)\cdot q_t(i)}{\sum_{i=1}^{n} p_t(i)\cdot q_0(i)}$$

Wahrscheinlichkeitsrechnung

Kombinatorik:

a) Permutationen $n!$ bzw. $\frac{\mathrm{n!}}{\mathrm{n_1!}\cdot n_2!\cdot\ldots\cdot n_k!}$

b) Kombinationen

	ohne Wiederholung	mit Wiederholung
mit Berücksichtigung der Anordnung	$\frac{n!}{(n-k)!}$	n^k
ohne Berücksichtigung der Anordnung	$\binom{n}{k}$	$\binom{n+k-1}{k}$

Bayes'sche Formel:

$$P(A_i \mid B) = \frac{P(B \mid A_i)\cdot P(A_i)}{\sum_j P(B \mid A_j)\cdot P(A_j)}$$

Verteilungen:

$BV(n;\pi)$: $\quad f(x\mid n;\pi)=\binom{n}{x}\cdot\pi^{x}\cdot(1-\pi)^{n-x}$ für $x=0,\ldots,n$

$HV(n;N;M)$: $\quad f(x\mid n;N;M)=\dfrac{\binom{M}{x}\cdot\binom{N-M}{n-x}}{\binom{N}{n}}$

für $x=\max\{0,n-(N-M)\},\ldots,\min\{n,M\}$

$PV(\lambda)$: $\quad f(x\mid\lambda)=\dfrac{\lambda^{x}}{x!}\cdot e^{-\lambda}$ für $x=0,1,2,\ldots$

$NV(\mu;\sigma)$: $\quad f(x\mid\mu;\sigma)=\dfrac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^{2}}$ für $-\infty<x<+\infty$

Erwartungswert und Varianz:

$$E(X)=\begin{cases}\sum_i x_i\cdot f(x_i) & \text{diskret}\\ \int x\cdot f(x)\,dx & \text{stetig}\end{cases}$$

$$V(X)=\begin{cases}\sum_i (x_i-E(X))^2\cdot f(x_i) & \text{diskret}\\ \int (x-E(X))^2\cdot f(x)\,dx & \text{stetig}\end{cases}$$

$$V(X)=E(X^2)-(E(X))^2$$

$$E(a+bX)=a+bE(X)$$
$$V(a+bX)=b^2\ V(X)$$
$$E(aX+bY)=a\ E(X)+b\ E(Y)$$
$$V(aX+bY)=a^2\ V(X)+b^2\ V(Y)+2ab\cdot(E(XY)-E(X)E(Y))$$
$$=a^2\ V(X)+b^2\ V(Y)+2ab\cdot COV(X,Y)$$

Endlichkeitskorrektur für Varianz:

$$\frac{N-n}{N-1}$$

Induktive Statistik

Konfidenzintervalle:

$$\left[\bar{X} - z_\alpha \cdot \frac{\sigma}{\sqrt{n}}; \bar{X} + z_\alpha \cdot \frac{\sigma}{\sqrt{n}}\right] \qquad \left[\bar{X} - t_\alpha \cdot \frac{\hat{\sigma}}{\sqrt{n}}; \bar{X} + t_\alpha \cdot \frac{\hat{\sigma}}{\sqrt{n}}\right]$$

$$\left[P - z_\alpha \cdot \sqrt{\frac{\pi(1-\pi)}{n}}; P + z_\alpha \cdot \sqrt{\frac{\pi(1-\pi)}{n}}\right]$$

$$\left[\frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}{\chi^2_{\left(1-\frac{\alpha}{2},\nu\right)}}, \frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}{\chi^2_{\left(\frac{\alpha}{2},\nu\right)}}\right] = \left[\frac{n \cdot S^2}{\chi^2_{\left(1-\frac{\alpha}{2},\nu\right)}}, \frac{n \cdot S^2}{\chi^2_{\left(\frac{\alpha}{2},\nu\right)}}\right] = \left[\frac{(n-1) \cdot S^2_{n-1}}{\chi^2_{\left(1-\frac{\alpha}{2},\nu\right)}}, \frac{(n-1) \cdot S^2_{n-1}}{\chi^2_{\left(\frac{\alpha}{2},\nu\right)}}\right]$$

Stichprobenumfang:

$$n = \frac{z_\alpha^2 \cdot \sigma^2}{e^2} \qquad n = \frac{z_\alpha^2 \widehat{\pi(1-\pi)}}{e^2}$$

Prüfgrößen:

$$z_r = \frac{\bar{x} - \mu_0}{\sqrt{\frac{\sigma^2}{n}}} \qquad z_r = \frac{p - \pi_0}{\sqrt{\frac{\pi_0 \cdot (1-\pi_0)}{n}}}$$

$$z_r = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \qquad z_r = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)_0}{\sqrt{\frac{\pi_1 \cdot (1-\pi_1)}{n_1} + \frac{\pi_2 \cdot (1-\pi_2)}{n_2}}}$$

$$\chi^2 = \sum_i \frac{(B_i - E_i)^2}{E_i} \qquad \chi^2_\sigma = \frac{\sum_i^n \left(X_i - \bar{X}\right)^2}{\sigma_0^2} = \frac{nS^2}{\sigma_0^2} = \frac{(n-1)S^2_{n-1}}{\sigma_0^2}$$

$$F = \frac{S^2_{n_1-1}}{S^2_{n_2-1}} \qquad F_{VA} = \frac{(n-r)S^2_{\text{ext}}}{(r-1)S^2_{\text{int}}} = \frac{S^2_{\text{ext}} / (r-1)}{S^2_{\text{int}} / (n-r)}$$

Tabelle 1
Tabelle der z-Werte (Standardnormalverteilung)

$\Phi(z) = P(-z \le Z \le z)$

z	Φ(z)	z	Φ(z)	z	Φ(z)
0.0	0.00000	1.4	0.83849	2.576	0.99000
0.1	0.07966	1.5	0.86639	2.6	0.99068
0.2	0.15852	1.6	0.89040	2.7	0.99307
0.3	0.23582	1.645	0.90000	2.8	0.99489
0.4	0.31084	1.7	0.91087	2.9	0.99627
0.5	0.38292	1.8	0.92814	3.0	0.99730
0.6	0.45149	1.9	0.94257	3.291	0.99900
0.7	0.51607	1.96	0.95000	3.5	0.99953
0.8	0.57629	2.0	0.95450	4.0	0.9999366
0.9	0.63188	2.1	0.96427	4.5	0.9999932
1.0	0.68269	2.2	0.97219	5.0	0.99999942
1.1	0.72867	2.3	0.97855	5.5	0.99999996
1.2	0.76986	2.4	0.98360	6.0	0.999999998
1.3	0.80640	2.5	0.98758		

Tabelle 2
Kritische Grenzen $x_{1-\alpha}$ für die χ^2-Verteilung mit ν Freiheitsgraden

(Spalten: $1-\alpha$ **; Zeilen:** ν **)** $P\left(X \le x_{1-\alpha}\right) = 1-\alpha$

	0.001	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99	0.999
1	0.00	0.00	0.00	0.00	0.02	2.71	3.84	5.02	6.63	10.80
2	0.00	0.02	0.05	0.10	0.21	4.61	5.99	7.38	9.21	13.80
3	0.02	0.11	0.22	0.35	0.58	6.25	7.81	9.35	11.40	16.30
4	0.09	0.30	0.48	0.71	1.06	7.78	9.49	11.10	13.30	18.50
5	0.21	0.55	0.83	1.15	1.61	9.24	11.10	12.80	15.10	20.50
6	0.38	0.87	1.24	1.63	2.20	10.60	12.60	14.40	16.80	22.40
7	0.60	1.24	1.69	2.17	2.83	12.00	14.10	16.00	18.50	24.30
8	0.86	1.65	2.18	2.73	3.49	13.40	15.50	17.50	20.10	26.10
9	1.15	2.09	2.70	3.33	4.17	14.70	16.90	19.00	21.70	27.90
10	1.48	2.56	3.25	3.94	4.86	16.00	18.30	20.50	23.20	29.60
100	61.92	70.06	74.22	77.93	82.36	118.50	124.30	129.60	135.80	149.40

Tabelle 3
Verteilungsfunktion der Poissonverteilung: $F(x)$ (Zeilen: $n\pi$; Spalten: x)

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0.1	.905	.995	.999											
0.2	.818	.982	.999											
0.3	.741	.963	.996	.999										
0.4	.670	.938	.992	.999										
0.5	.606	.910	.986	.998	.999									
0.6	.548	.878	.977	.997	.999									
0.7	.497	.844	.966	.994	.999									
0.8	.449	.809	.952	.991	.999									
0.9	.416	.773	.937	.987	.998	.999								
1.0	.368	.736	.919	.981	.996	.999								
2.0	.135	.406	.677	.857	.947	.983	.996	.999						
3.0	.050	.199	.423	.647	.815	.916	.966	.988	.996	.999				
4.0	.018	.092	.238	.434	.629	.785	.889	.950	.979	.992	.997	.999		
5.0	.007	.040	.125	.265	.441	.616	.762	.867	.932	.968	.986	.995	.999	
6.0	.003	.017	.062	.151	.285	.446	.606	.744	.847	.916	.957	.980	.991	.996
7.0	.001	.007	.030	.082	.173	.300	.447	.590	.721	.836	.907	.943	.975	.982
8.0		.003	.014	.042	.100	.191	.313	.453	.592	.717	.816	.888	.936	.966
9.0		.002	.006	.021	.055	.115	.207	.370	.453	.581	.704	.801	.878	.928
10.0		.001	.003	.010	.029	.067	.130	.220	.333	.458	.583	.697	.792	.865
20.0								.001	.002	.005	.011	.021	.039	.066

Tabelle 4
0.95-Fraktile $x_{0.95;\nu_1,\nu_2}$ der $FV(\nu_1,\nu_2)$ (Zeilen: ν_2; Spalten: ν_1)

$$P\left(X \le x_{0.95;\nu_1,\nu_2}\right) = 0.95 \quad x_{\alpha;\nu_1,\nu_2} = \frac{1}{x_{1-\alpha;\nu_2,\nu_1}}$$

	1	2	3	4	5	6	7	8	9	10	20	50	100
1	161.0	200.0	216.0	225.0	230.0	234.0	237.0	239.0	241.0	242.0	248.0	252.0	254.0
2	18.5	19.0	19.2	19.3	19.3	19.3	19.4	19.4	19.4	19.4	19.5	19.5	19.5
3	10.1	9.6	9.3	9.1	9.0	8.9	8.9	8.9	8.8	8.7	8.7	8.6	8.6
4	7.7	7.0	6.6	6.4	6.3	6.2	6.1	6.0	6.0	6.0	5.8	5.7	5.7
5	6.6	5.8	5.4	5.2	5.1	5.0	4.9	4.8	4.8	4.7	4.6	4.4	4.4
6	6.0	5.1	4.8	4.5	4.4	4.3	4.2	4.2	4.1	4.1	3.9	3.8	3.7
7	5.6	4.7	4.4	4.1	4.0	3.9	3.8	3.7	3.7	3.6	3.4	3.3	3.3
8	5.3	4.5	4.1	3.8	3.7	3.6	3.5	3.4	3.4	3.4	3.2	3.0	3.0
9	5.1	4.3	3.9	3.6	3.5	3.4	3.3	3.2	3.2	3.1	2.9	2.8	2.8
10	5.0	4.1	3.7	3.5	3.3	3.2	3.1	3.1	3.0	3.0	2.8	2.6	2.6
15	4.6	3.7	3.3	3.0	2.9	2.8	2.7	2.6	2.6	2.5	2.3	2.2	2.1
20	4.4	3.5	3.1	2.9	2.7	2.6	2.5	2.5	2.4	2.4	2.1	2.0	1.9
25	4.2	3.4	3.0	2.8	2.6	2.5	2.4	2.3	2.3	2.2	2.0	1.8	1.8
30	4.2	3.3	2.9	2.7	2.5	2.4	2.3	2.3	2.2	2.2	1.9	1.8	1.7
50	4.0	3.2	2.8	2.6	2.4	2.3	2.2	2.1	2.1	2.0	1.8	1.6	1.5
100	3.9	3.1	2.7	2.5	2.3	2.2	2.1	2.0	2.0	1.9	1.7	1.5	1.4
200	3.9	3.0	2.7	2.4	2.3	2.1	2.1	2.0	1.9	1.9	1.6	1.4	1.3

Tabelle 5
Ausgewählte t-Werte für die Student-Verteilung
$P(-t \le X \le t) = 1 - \alpha$ **(Zeilen: Freiheitsgrade, Spalten: α)**

	α					α			
	0.10	0.05	0.01	0.001		0.10	0.05	0.01	0.001
1	6.31	12.71	63.66	636.37	11	1.80	2.20	3.11	4.44
2	2.92	4.30	9.93	31.60	12	1.78	2.18	3.06	4.32
3	2.35	3.18	5.84	12.92	13	1.77	2.18	3.01	4.22
4	2.13	2.78	4.60	8.61	14	1.76	2.15	2.98	4.14
5	2.02	2.57	4.03	6.87	15	1.75	2.13	2.95	4.07
6	1.94	2.45	3.71	5.96	20	1.73	2.09	2.85	3.85
7	1.90	2.37	3.50	5.41	25	1.71	2.06	2.79	3.73
8	1.86	2.31	3.36	5.04	30	1.70	2.04	2.75	3.65
9	1.83	2.26	3.25	4.78	50	1.68	2.01	2.68	3.50
10	1.81	2.23	3.17	4.59	100	1.66	1.98	2.63	3.39