UNIVERSITÄT PADERBORN
Die Universität der Informationsgesellschaft

Multiple Criteria Decision Making

Comparison of ELECTRE and AHP (Analytical Hierarchy Process)
Evaluation of the express mail delivery companies (UPS, DHL…)

15.05.2008

University of Paderborn

Team 2: Christian Kipp, Harry Kran, Annette Bösherz, Sebastian Schweer, Henning Galicki

Contents

Exercise 1 ..... - 3 -

1. Brief Introduction to MCDM ..... - 3 -
2. Some Test Criteria for Evaluating MCDM Methods ..... - 3 -
3. ELECTRE Method ..... - 3 -
   a. Introduction ..... - 3 -
   b. Modeling Preferences using an outranking relation ..... - 4 -
   c. Structure of ELECTRE Methods ..... - 4 -
   d. A short description of ELECTRE Method ..... - 4 -
4. AHP (Analytical Hierarchy Process) ..... - 5 -
   a. Introduction ..... - 5 -
   b. Method ..... - 5 -
   c. Review ..... - 6 -
5. Comparison ..... - 6 -

Exercise 2 ..... - 7 -

Evaluation of the express mail delivery companies (UPS, DHL, …) ..... - 7 -

1. Description of the specific problem ..... - 7 -
2. Variants ..... - 7 -
3. Consistent Family of Criteria ..... - 7 -
4. Evaluation matrix ..... - 8 -
5. Model of the DM's preferences ..... - 8 -
6. Data for ELECTRE lll ..... - 9 -
   a. Criteria Table ..... - 9 -
   b. Alternatives Table ..... - 9 -
   c. Performance Table ..... - 10 -
   d. Thresholds Table ..... - 10 -
7. Results (calculated by ELECTRE lll) ..... - 11 -
   a. Ranks in final Preorder ..... - 11 -
   b. Ranking Matrix ..... - 11 -
   c. Credibility Matrix ..... - 11 -
   d. Distillations ..... - 12 -
   e. Final Graph ..... - 12 -
8. Conclusion ..... - 13 -

References ..... - 13 -

Exercise 1

1. Brief Introduction to MCDM

A typical MCDM problem is concerned with the task of ranking a finite number of decision alternatives/actions, each of which is explicitly described in terms of different characteristics (also often called attributes, decision criteria, or objectives) which have to be taken into account simultaneously. Usually, the performance values a_{ij} and the criteria weights w_j are viewed as the entries of a *decision matrix* as shown below. The a_{ij} element of the decision matrix represents the performance value of the *i-th* alternative in terms of the *j-th* criterion. The w_j value represents the weight of the *j-th* criterion [WANG1].

A is a set of Alternatives. A set **A** is a collection of objects, candidates, variants, decisions that are to be analyzed and evaluated during the decision process.
C is a consistent family of Criteria, a set of functions [ZAK1].

Problem: $\max\{a_1,...,a_n : a_i \in A\}$

	Criteria			
	C_1	C_2	...	C_n
Alternatives	$(w_1$	w_2	...	$w_n)$
A_1	a_{11}	a_{12}	...	a_{1n}
A_2	a_{21}	a_{22}	...	a_{2n}
...	...	...	...	...
A_m	a_{m1}	a_{m2}	...	a_{mn}

Table 1: *Evaluation Table*

DOMINANCE RELATION
Given two elements A_i and A_j of A. A_i dominates A_j (A_i D A_j) if $a_{iw}(A_i) \geq a_{jw}(A_j)$ $w = 1,..,n$

EFFICIENT (PARETO-OPTIMAL) ALTERNATIVE
Alternative A_i is efficient, if no alternative dominates it

2. Some Test Criteria for Evaluating MCDM Methods

In [TRIAN1], three test criteria were established to evaluate the performance of MCDM methods by testing the validity of their ranking results. These test criteria are as follows:

Test Criterion #1:
An effective MCDM method should not change the indication of the best alternative when a non-optimal alternative is replaced by another worse alternative (given that the relative importance of each decision criterion remains unchanged).

Test Criterion #2:
The rankings of alternatives by an effective MCDM method should follow the transitivity property.

Test Criterion #3:
For the same decision problem and when using the same MCDM method, after combining the rankings of the smaller problems that an MCDM problem is decomposed into, the new overall ranking of the alternatives should be identical to the original overall ranking of the undecomposed problem.

3. ELECTRE Method

a. Introduction

The acronym ELECTRE stands for: ELimination and Choice Expressing the Reality.
The main idea of this method is the proper utilization of what is called “outranking relations” to rank a set of alternatives. [Wang1]

Context in which ELECTRE methods are relevant:

- The Decision Maker (DM) wants to include at least three criteria in the model
- Actions are evaluated (for at least one criterion) on an ordinal scale. These scales are not suitable for the comparison of differences.
- A strong heterogeneity related with the nature of evaluations exists among criteria (duration, noise, distance,...). This makes it difficult to aggregate all the criteria in a unique and common scale.
- For at least one criterion the following holds true: small differences of evaluations are not significant in terms of preferences. While the accumulation of several small differences may become significant. This requires the introduction of discrimination thresholds $(I \text{ and } P)$ which leads to a preference structure with a comprehensive intransitive indifference binary relation.

b. Modeling Preferences using an outranking relation

Preferences in ELECTRE methods are modeled by using binary outranking relations, S, whose meaning is "at least as good as". Considering two actions a and b, four situations may occur:

- aSb and not bSa, i. e. aPb (a is strictly preferred to b)
- bSa and not aSb, i. e. bPa (b is strictly preferred to a)
- aSb and bSa, i. e. aIb (a is indifferent to b)
- not aSb and not bSa, i. e. aRb (a is incomparable to b)

Given a binary relation on set *A* it is extremely helpful to build a graph *G* = (*V,U*), where *V* is the set of vertices and *U* the set of arcs. For each action $a \in A$ we associate a vertex $i \in V$ and for each pair of actions $(a,b) \in A$ the arc (i,l) exists either if aPb or aIb. An action a outranks b if and only if the arc (i,l) exists. If there is no arc between vertices i and l it means that a and b are incomparable; if two reversal arcs exist, there is an indifference between both a and b [SPR1].

Outranking relation is a binary relation S defined in A, such that aSb if, there are enough arguments to decide that a is at least as good as b. Outranking relation S is a sum of the indifference I and preference P relations: $S = P \cup I$ [ZAK1]

c. Structure of ELECTRE Methods

ELECTRE methods comprise two main procedures: construction of one or several outranking relation(s) followed by an exploitation procedure. The construction of one or several outranking relations(s) aims at comparing in a comprehensive way each pair of actions. The exploitation procedure is used to elaborate recommendations from the results obtained in the first phase. The nature of the recommendations depends on the problematic (choosing, ranking or sorting). Hence, each method is characterized by its construction and its exploitation procedures.

d. A short description of ELECTRE Method

A comprehensive treatment of ELECTRE methods may be found in the books by B. Roy and D. Bouyssou [ROY1].

Choice Problematic

The objective of this problematic consists of aiding DMs in selecting a way that a single action may finally be chosen, explicit to determine a subset of actions considered to be the best with respect to F.
ELECTRE I: The method is very simple and it should be applied only when all the criteria have been coded in numerical scales with identical ranges. In such a situation we can assert that an action "a outranks b" - aSb.

Mathematical formulation: [SPR1]

$\sum_{j \in J} w_j = 1$ where J — is the set of the indices of the criteria

$c(aSb) = \sum_{\{j: g_j(a) \geq g_j(b)\}} w_j$ — concordance index

$d(aSb) = \max_{\{j: g_j(a) < g_j(b)\}} \{g_j(b) - g_j(a)\}$ — discordance index

Both concordance and discordance indices have to be computed for every pair of actions (a,b) in the set A, where $a \neq b$

Ranking Problematic

Divide A into subsets according to some norms. Here we are concerned with the ranking of all the actions belonging to a given set of actions from the best to the worst. ELECTRE II was the first method especially designed to deal with ranking problems.
ELECTRE II: Now there are two embedded relations: a strong outranking relation followed by a weak outranking relation. Both the strong and weak relations are built thanks to the definition of two concordance levels, $s^1 > s^2$, where $s^1, s^2 \in \left[0.5, 1 - \min_{j \in J} w_j\right]$. Now the assertion "a outranks b" can be defined as follows: $c(aSb) \geq s^r$ and $c(aSb) \geq c(bSa)$, *for* $r = 1,2$

ELECTRE III: Here the outranking relation can be interpreted as a fuzzy relation. The novelty of this method is the introduction of pseudo-criteria instead of true-criteria.

Sorting Problematic

The objective of sorting problems is to rank the actions of A from best to worst. Therefore a set of categories must be *a priori* defined.
ELECTRE TRI: Here the categories are ordered. $C = \{C_1, ..., C_k\}$ denote the set of categories. The assignment of a given action a to a certain category C_h results from the comparison of a to the profiles defining the lower and upper limits of the categories

4. AHP (Analytical Hierarchy Process)

a. Introduction

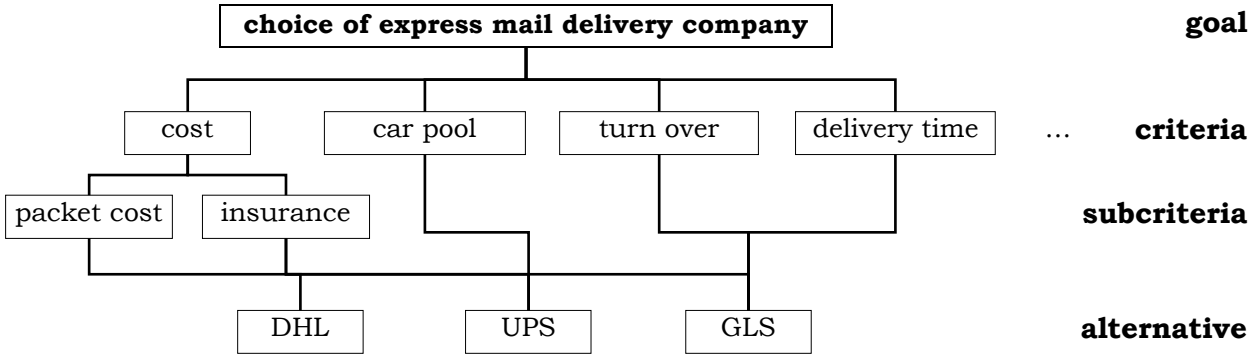
AHP method belongs to the field of MCDM (Multi Criteria Decision Making) as well and is the abbreviation for Analytical Hierarchy Process. It belongs to the group of problems, which should help to rank a number of alternatives and take different criteria into account simultaneously [WANG1]. It was developed by Thomas Saaty.

b. Method

AHP method and the model of preferences are based on pairwise comparison.
But first let us gain a better overview of this method through describing its steps [HUN1].

1. Decompose the problem into a hierarchical structure
2. Perform judgements to establish priorities for the elements of the hierarchy
3. synthesis of the model
4. Perform a sensitivity analysis

(1) The basic structure of an AHP method consists of different elements: goal, criteria and alternatives. The goal is the overall destination we want to achieve during the modeling process. Criteria are a kind of characterization of elements e.g. certain attributes. These criteria lead to different alternatives, of which we can choose the best one for our goal. For a better understanding we created a small schematic graph [REICH1] based on the problem given in task 2)



(2) The next step is now to judge and to set priorities within the elements of the hierarchy. Obviously this is restricted to qualitative kind of information/elements. Quantitative information/elements are ranked in a natural way. These judgements are made by the decision maker (DM). It is done through pairwise comparison of all elements and through setting values for two elements. In detail this value is the relative importance between two elements.

The pairwise comparison can be written down in matrix of this form:

$$A = \begin{pmatrix} 1 & a_{12} & \dots & a_{1n} \\ \frac{1}{a_{12}} & 1 & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{1}{a_{1n}} & \frac{1}{a_{2n}} & \dots & 1 \end{pmatrix}$$

a_{ij} is defined as $a_{ij} = \frac{w_i}{w_j}$. The weights w are taken from the Fundamental Scale [HUN1]:

Definition	Intensity of preferences
equally important	**1**
moderately more important	**3**
strongly more important	**5**
very strongly more important	**7**
extremely more important	**9**
interim values	**2,4,6,8**

(3) After the judgments, all the elements are synthesized by the help of a mathematic model. The aim is to find out inconsistencies within the matrix. For Example:
- Criteria A is "two" times more important than criteria B
- Criteria B is "three" times more important than criteria C
- Criteria A is "four" times more important than criteria C

The last statement is wrong because of the first two statements. Correct is:
➔ Criteria A is "six" times more important than criteria C, because of transitivity

(4) Perform a sensitive analyze in order to look at the results when trying different criteria weights. It helps to gain borders between different results

c. Review

All in all AHP is a common and simple method for MCDM. It can even be applied by using a spreadsheet program like Excel.
Quantitative as well as qualitative information can be taken into account.

Some negative accepts are:
- subjective view within the pairwise comparisons
- easy appearance of inconsistence
- unique 9 point scale, therefore hard to compare with other methods e.g. ELECTRE

But overall these problems can be solved if you have a qualified and experienced Decision Maker

5. Comparison

Manner of Synthesizing (aggregating) the DM's global preferences multiobjective methods based on the utility function (AHP, UTA, …)
Manner of Synthesizing (aggregating) the DM's global preferences multiobjective methods based on the outranking relation (ELECTRE, promethee) [ZAK1]

Compared with the simple process and precise data requirement of the AHP methods, ELECTRE methods are able to apply more complicated algorithms to deal with the complex and imprecise information from the decision problems and use these algorithms to rank the alternatives [Wang1].

Modeling of DM's preferences

- Electre method utilized weights of criteria to express the DM's opinion about the importance of particular parameters and thresholds of indifference (q), preference (p) and veto (v) for each criterion to express the DM's sensitivity on the changes of their values; [ZAK2]
- AHP method utilizes relative weights on each level of hierarchy, which means that pairwise comparisons are carried out to define relative importance (advantage) of one element (variant, criterion, subcriterion) against the others [ZAK2].

Electre and AHP methods preference models were appreciated. Positive opinions about Electre and AHP were expressed by 78% and 74% of the surveyed persons. DMs declared that those models are easy to understand, although there were some opinions suggesting that the meaning of veto threshold in Electre method in not very clear to DMs. Furthermore relative comparison between objects in AHP induces certain difficulties.

Electre and AHP methods are the most reliable and users' friendly MCDA methods; the models of preferences proposal in those methods and final rankings generated by them are highly appreciated;

Exercise 2

Evaluation of the express mail delivery companies (UPS, DHL, …)

1. Description of the specific problem

We play the role of a representative of a company that is not satisfied with its current express mail deliverer and wants to select a new one. For our exercise we have selected a typical situation in our daily business to send a customer's order into one of the Major Cities in the European Union. As instance we have selected an international delivery from Germany to Stockholm (Sweden) and referred to a package of 10 kg, dimensions: (length, breadth, depth) 40*20*10 cm and package value of 1000,-€, with pickup service.
The mail-deliverer are characterized by the following components: Founding year (→ market experience), number of employees (the more employees the faster/more job can be done), number of depots (reachability), number of delivered packages per year, turnover, fleet (the more vehicles, the more jobs can be done), delivery time (promised by deliverer), delivery costs (as per tariff) and till what value the insurance is inclusive. These components are important for us and have to be compared between the companies. In support of our comparisons and decision making we use the system ELECTRE lll which requires the following necessary data.

2. Variants

1. Hermes PaketService
2. DHL
3. UPS
4. DPD
5. GLS
6. FedEx
7. TNT

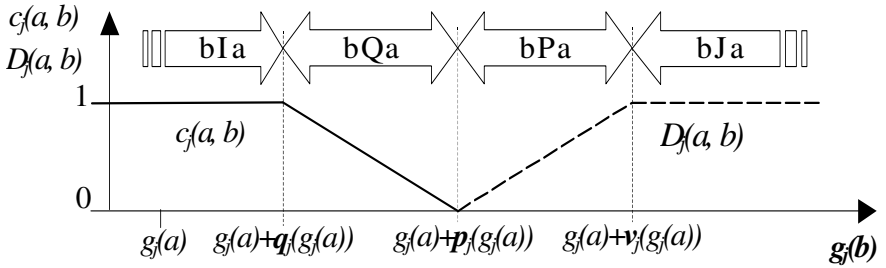
3. Consistent Family of Criteria

C1 – founding year [year], minimize
C2 – number of employees [number], maximize
C3 – number of depots [number], maximize
C4 – number of delivered packages per year [number in million], maximize
C5 – turnover in year 2007 [€ in billion], maximize
C6 – number of fleet [number], maximize
C7 – delivery time [days], minimize
C8 – delivery costs [€], minimize
C9 – insurance incl. till value [€], maximize

4. Evaluation matrix

International	founding year (year)	number of employees (number)	number of depots (number)	number of delivered pack./year	turnover in bn. €	fleet (number)	delivery time (days)	delivery costs €	insurance free till (value)
Hermes Paket Service	1972	13.000	115	235 Mio.	0,59	2000	5	14,90	500,00
DHL	1969	124.000	450	1500 Mio.	13,87	76.000	4	22,00	500,00
UPS	1907	425.300	1800	4000 Mio.	25,81	93.637	1	190,00	1000
DPD	1976	22.000	500	730 Mio.	3,17	15.000	2	18,80	520,00
GLS	1989	220.000	650	311 Mio.	1,6	17.800	2	20,80	750,00
FedEx	1973	240.000	1401	1190 Mio.	22,0	42.000	1	92	0
TNT	1946	48.000	2331	228 Mio.	6,55	26760	2	24,80	2500

USD : Euro = 1,60

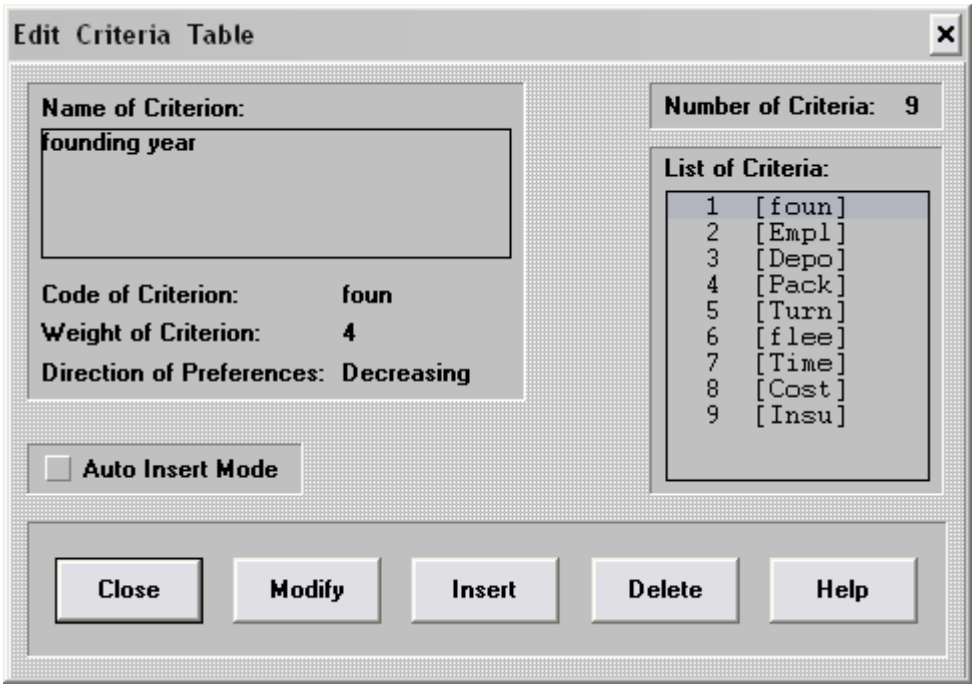


5. Model of the DM's preferences

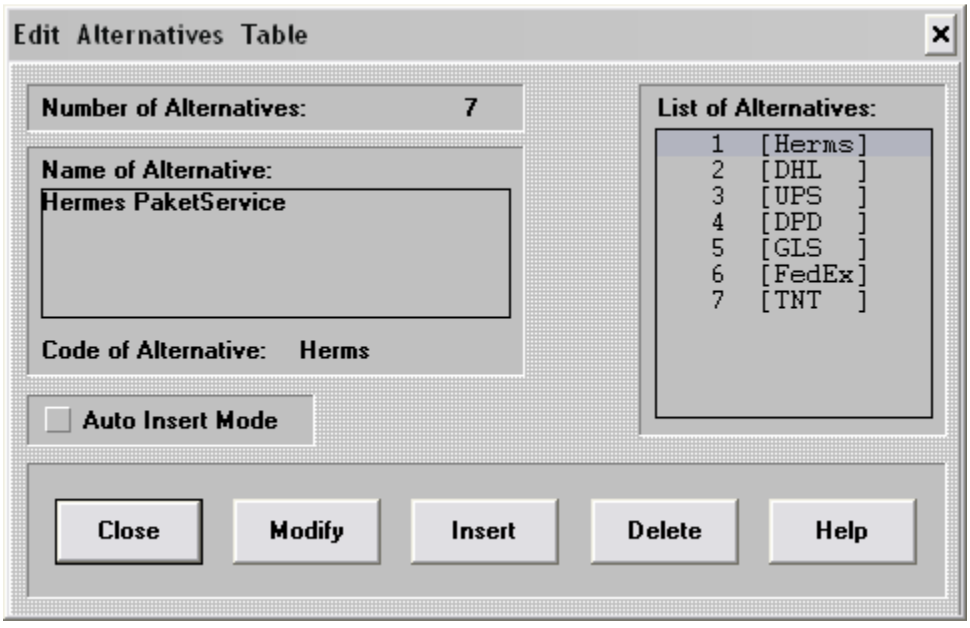
CRITERIA	UNIT	q_j	p_j	v_j	w_j	kp_j	
1. founding year	year	1900	1910	1950	4	Min	**inverse**
2. number of employees	number	15000	100000	450000	9	Max	**direct**
3. number of depots	number	130	500	2500	7	Max	**direct**
4. number of delivered pack. a day	number	300	1000	4000	6	Max	**direct**
5. turnover	Mrd. € in 2007	1	5	18	5	Max	**direct**
6. fleet	number	2800	40000	90000	10	Max	**direct**
7. delivery time	days	1	2	3	8	Min	**inverse**
8. delivery costs	€	5	10	20	10	Min	**inverse**
9. insurance free until (value)	€	500	900	2000	6	Max	**direct**

6. Data for ELECTRE lll

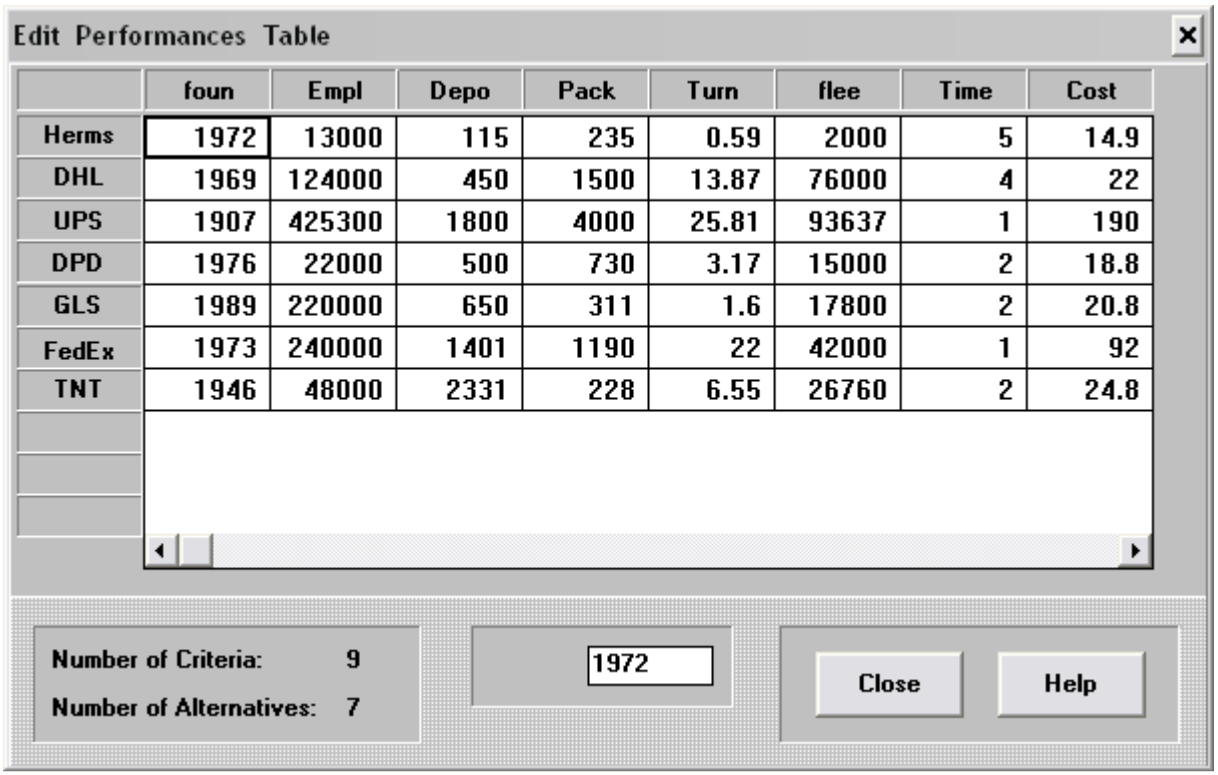
a. Criteria Table



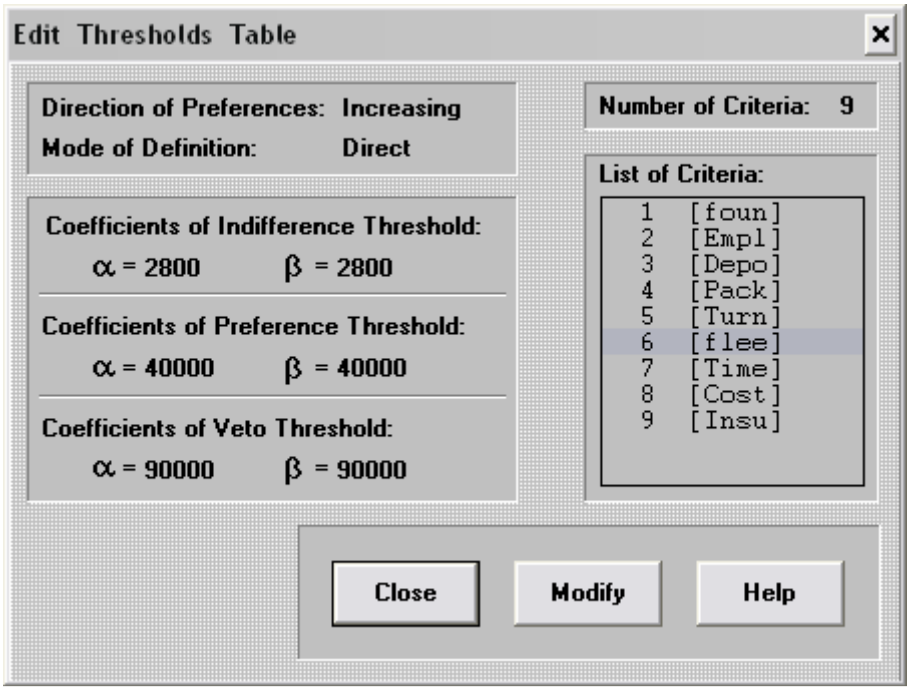
b. Alternatives Table



c. Performance Table



d. Thresholds Table



7. Results (calculated by ELECTRE lll)

a. Ranks in final Preorder

Ranks in Final Preorder

Rank	Alternative
1	TNT
2	UPS FedEx
3	DHL DPD GLS
4	Herms

b. Ranking Matrix

Ranking Matrix

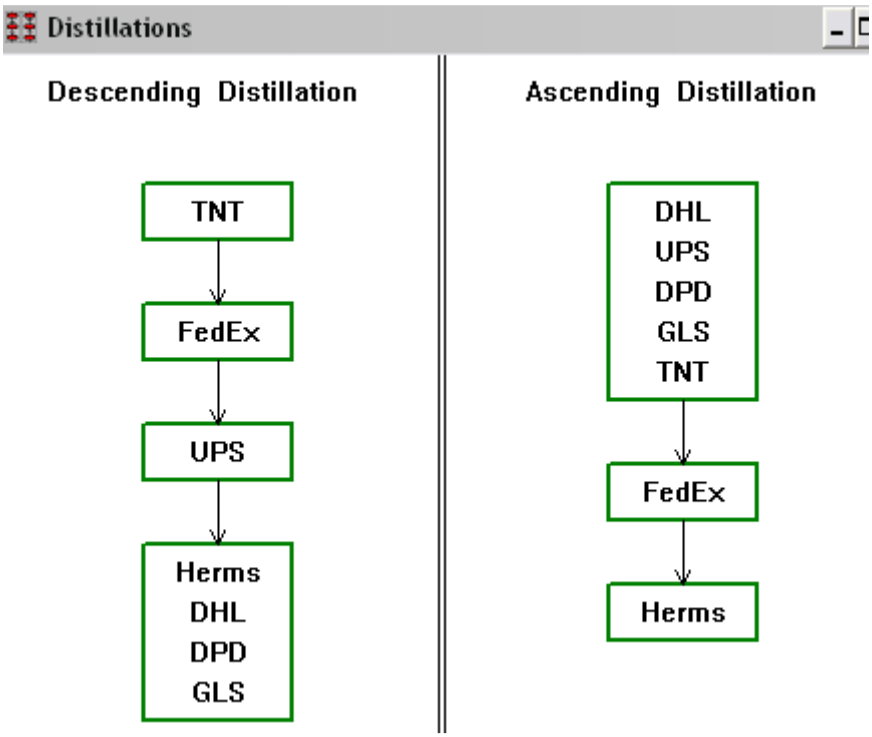
	Herms	DHL	UPS	DPD	GLS	FedEx	TNT
Herms	I	P⁻	P⁻	P⁻	P⁻	P⁻	P⁻
DHL	P	I	P⁻	I	I	R	P⁻
UPS	P	P	I	P	P	R	P⁻
DPD	P	I	P⁻	I	I	R	P⁻
GLS	P	I	P⁻	I	I	R	P⁻
FedEx	P	R	R	R	R	I	P⁻
TNT	P	P	P	P	P	P	I

c. Credibility Matrix

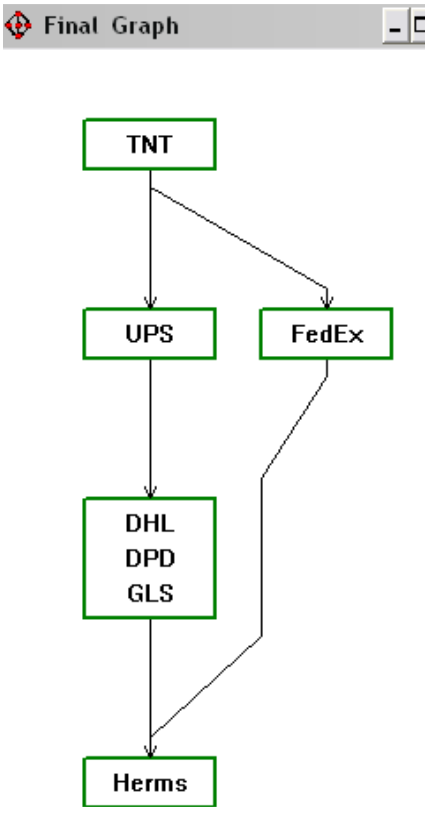
Credibility Matrix

	Herms	DHL	UPS	DPD	GLS	FedEx	TNT
Herms	1	0.92	0.66	0.99	1	0.8	0.95
DHL	1	1	0.93	1	1	0.93	1
UPS	0.85	0.91	1	0.87	0.89	1	0.85
DPD	1	0.97	0.92	1	1	0.93	1
GLS	1	0.93	0.92	1	1	0.92	0.98
FedEx	1	1	0.91	1	0.94	1	0
TNT	1	1	0.97	1	1	0.98	1

d. Distillations



e. Final Graph



8. Conclusion

During the research into the market of express-mail delivery companies a lot of data has been found, which sometimes wasn't easy to find or wasn't consistent among different sources. Therefore some data/criteria have been left out and consequently an absolutely correct data base can not be guaranteed.
All found data have been filled in the forms of ELECTRE lll as performances of the different alternatives/companies. All selected criteria was weighted with a number 1 to 10 (1 for unimportant and 10 for very important).
The definition of the model of DM's preferences provides the most import valuation, where the preferences of the Decision Makers are appointed.
As the results show, one express mail deliverer is on the top of the favoured companies ($\rightarrow$TNT). Its advantages among other things are the relative low delivery price, the highest number of depots and the relative high number of fleet. Although UPS has many good cases it was displaced to the next lower level. Whereas a small modifying of the preferences of delivery costs or the weight of costs (from 10 to 9), would affect that UPS then will spearhead with TNT.
In conclusion the considered favourite is TNT and a contract between the seeking company and the express-mail company can be worked out. In the process maybe yet better conditions can be bargained.

References

EXERCISE I

[HUN1] Tihomir Hunjak (1997), *Mathematical foundations of the methods for multicriterial decision making* Department of Mathematics, Faculty of Organisation and Informatics, Pavlinska, Varazdin, Croatia

[REICH1] Thomas Reichhard, *Mehrkriterielle Entscheidungen mit dem AHP Verfahren,* seminar paper

[ROY1] B. Roy and D. Bouyssou (1993), *Aide Multicritère à la Décision: Méthodes et Cas. Economica, Paris*

[SPR1] Springer's International Series, edited by José Figueira – University of Coimbra, Salvatore Grece – University of Catania, Matthias Ehrgott – University of Auckland (2005), *Multiple Criteria Decision Analysis: State of the art surveys* [Chapter 4, Chapter 9]

[TRIAN1] Triantaphyllou, E. (2000), *Multi-Criteria Decision Making Methods: A Comparative Study*, Kluwer Academic Publishers, Boston, MA, U.S.A.

[WANG1] Xiaoting Wang (2007), *Study of ranking irregularities when evaluating alternatives by using some ELECTRE methods and a proposed new MCDM method based on regret and rejoicing,* Louisiana State University and Agricultural and Mechanical College

[ZAK1] Jacek Zak (2007) – academic tour, *Multiple Criteria Decision Making in business applications*

[ZAK2] Jacek Zak, *The comparison of multiobjective ranking methods applied to solve the mass transit systems' decision problems*

EXERCISE II

General Information
http://www.posttip.de/Paketdienste.html

Hermes Logistik Gruppe
http://www.hermes-logistik-gruppe.de/
Prices: *http://privatpaketservice.hlg.de/wps/portal/PRIPS_DEU/PREISE*
Otto Group (Parent Company of Hermes):
http://www.ottogroup.com/uploads/media/Otto_Group_GB_06_07_dt_72_rgb.pdf
Source for number of vehicles:
http://www.volkswagen-group-fleet.de/aktuelles/nachrichten/alle-nachrichten/nachrichten-details/article/500-crafter-fuer-hermes-logistik.html?tx_ttnews%5BbackPid%5D=89

DHL
http://www.dhl.de; Preise: http://www.dhl.de/preise;
Deutsche Post (Parent Company) Annual Report 2007:
http://investors.dpwn.de/de/investoren/publikationen/archiv/2007/finanzpublikationen/dpwn_annual_report22007_de.pdf

UPS
Facts 2007:
http://www.ups.com/content/us/en/about/facts/worldwide.html
http://www.pressroom.ups.com/mediakits/factsheet/0,2305,866,00.html
http://www.ups.com/; Prices: see pay scale table

DPD
http://www.dpd.net/; Prices: see pay scale table

GLS
http://www.gls-germany.com;
Prices: *http://www.gls-germany.com/de/shop/preisklassen.php3*

FedEx
http://www.fedex.com
Annual Report 2007
http://www.fedex.com/us/investorrelations/downloads/annualreport/2007annualreport.pdf
http://news.van.fedex.com/fedexexpress

TNT
Annual Report 2007:
http://group.tnt.com/annualreports/annualreport07/downloads/tnt-annual-report-2007-chapter03.pdf